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(Nota interna: n. 247)

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1) - It was pointed out by Weinberg that strangeness-conserving weak currents may be classified into two classes according to their properties under $G = Ce^{i\pi I_2}$ (1). The recently confirmed hypothesis of a conserved vector current(2) implies that the vector current is entirely of first class. At present there is however no definite evidence against second class terms in the axial current. It would be extremely useful to have a proof of the non-existence of such terms. In simplest theories of weak interactions based on unitary symmetry one expects only first class currents(3). As remarked by Weinberg(1), the rigorous absence (up to electromagnetic corrections) of second class terms would strongly suggest a relation between weak currents and isotopic spin.

In this note we examine the contribution from a possible second class axial current to radiative muon absorption by nuclei. This process is very sensitive to the presence of a second class axial term. It was already noted by Weinberg(1) that the comparison of mirror β-transitions, like B^{12}, $N^{12} \rightarrow C^{12}$ ($\Delta J = 1$, no) or B^8, Li^8 ($\Delta J = 0$, no), could provide a test for second class currents. Quite similarly, an inequality of the rates for $\Sigma^{\pm} \rightarrow \Lambda^0 + e^{\pm} + \nu$ would require second class currents. The present data on these decays are consistent with equal rates of about $(4 \pm 2) \times 10^5\ sec^{-1}$; the large experimental error does not allow for any conclusion.

Recent measurements have shown that the ft values of B^{12} and N^{12} differ by 14 $\pm$ 2,5%[4] or by 16 $\pm$ 3%[5]. Huffaker and Greuling[6] on the assumption that the nuclear matrix elements of the two transitions are equal, have derived a relation that must be satisfied by the pseudoscalar term and the possible second class tensor term of the axial weak current. Well-known theoretical considerations suggest a value $\mu C_P \cong 8 C_A$ for the pseudoscalar constant of muon absorption as compared to the axial constant C_A (μ = muon mass)[7]. A separate determination of C_P and C_T (the constant of the possible tensor term) is obtained by Huffaker and Greuling by extending the analysis by Morita and Fujii[8] of μ^- capture in C^{12} (resulting in B^{12} in its ground state) by including C_T. With a value of 1.14 for the B^{12} - N^{12} ft ratio and a value of 6.7 x 10^3 sec^{-1} for the μ^- capture rate in C^{12}[9] they obtain two sets of solutions: (i) $\mu C_P = 26 C_A$, $\mu C_T = 3.4\,(\mu/m)\,C_A$; and (ii) $\mu C_P = 35\,C_A$, $\mu C_T = 3.4\,(\mu/m)\,C_A$.

The result of the CERN experiment on radiative μ^--absorption in Ca^{40}[10] does not agree with both sets (i) and (ii) as they would lead to radiative rates larger than the measured rate.

Conversi et al.[10] give a ratio, R, of radiative to ordinary absorption in Ca^{40} of 3,1 $\pm$ 0,6. The value of R is obtained by extrapolation from the measured high energy tail of the photon spectrum. The spectral shapes do not very appreciably for different choices of the constants.

It must be remarked that the determination of C_T by Huffaker and Greuling[6] depends mostly on the ratio of ft(N^{12}) to ft(B^{12}). However the assumption that the mirror matrix elements are equal makes their conclusion not very firm. The size of the Coulomb effects, which would make the mirror matrix elements different, are very difficult to calculate. On the other hand, Weinberg estimates Coulomb effects as possibly causing a 1% inequality in the ft values, using known values of isotopic spin impurities in light nuclei[1].

In this note we also interpret the CERN results on radiative muon absorption from the following standpoint: we assume that $\mu C_P = 8C_A$ as suggested theoretically[7] and try to determine C_T from the radiative branching ratio. This analysis favors a value of $C_T \gtrsim C_A$ and seems to disagree with $C_T = 0$.

In this connection a relatively large C_T would be helpful, from the same standpoint, in lowering the predictions for the μ^- absorption rate in liquid hydrogen[11], as seems to be required to fit the hydrogen data, which depend on largely unknown molecular parameters. We do not feel however that our interpretation of the radiative absorption data provides a definite evidence for a second class term in the axial current. In particular we have assumed $\mu C_P = 8C_A$ only from theoretical considerations[7]; the calculations also depend inevitably from the nuclear dynamics and, recently, some basic assumptions usually made in calculating nuclear μ-absorption have been criticized[12].

We suggest however that inclusion of C_T in the analysis of both radiative and non-radiative μ-absorption should be considered in future more refined experiments. The parameters of the muon-absorption interaction are not well known ar present. Recent analyses of muon absorption in He^3 are confortably consistent with $\mu C_P = 8C_A$[13]; other analyses of muon data seem to disagree with simple explanations[14]. Study of particular energetic β-transitions, such as the 10.40 MeV β-spectrum of N^{16}[6], could also be very useful in deciding as to the existence of an induced tensor term in the axial current.

2) - We write the effective hamiltonian for $\mu^- + p \rightarrow n + \nu$ as

$$H_{eff} = \left[\bar{n}(C_A\gamma_5\gamma_\lambda + C_P\gamma_5 q_\lambda - iC_T\gamma_5\,\sigma_{\lambda\mu}q_\mu + C_V\gamma_\lambda - iC_M\,\sigma_{\lambda\mu}q_\mu)p \right] \times$$
$$\times\,(\bar{\nu}(1-\gamma_5)\,\gamma_\lambda\,\mu) \qquad (1)$$

where: $q_\lambda = (p - n)_\lambda$ is the difference of the proton and neutron momenta; the form factors C are real functions of q (wel assume time reversal); n, p, ν and μ also denote the fields of the fermions. The vector part of H is the most general one consisting with the conserved vector current hypothesis[15]; the axial part includes a term proportional to C_T which takes on under G-conjugation a sign opposite to that of the terms proportional to C_A and C_P. In the following we shall neglect the q^2 dependence of C_A, C_T, C_V and C_M and assume a dependence of the form $(m_\pi^2 - q^2)^{-1}$ for C_P (dominance of one-pion exchange)[7].

For the radiative process $\mu^- + p \rightarrow n + \nu + \gamma$ we use the following model:

(i) The radiative amplitudes proportional to C_A, C_T, C_V and C_M are calculated from bremsstrahlung graphs, with the photon emitted from the external muon line, from the external neutron line (with coupling $-\mu_n \sigma_{\mu\nu} F_{\mu\nu}$), and from the external proton line (with coupling $e\gamma_\mu A_\mu - \mu_p \sigma_{\mu\nu} F_{\mu\nu}$); and from catastrophic graphs obtained through the substitution $q_\lambda \rightarrow q_\lambda - eA_\lambda$ in the terms proportional to C_A, C_T, C_V and C_M in the effective hamiltonian (1).

(ii) The radiative amplitudes proportional to C_P are calculated by fully taking into account the one-pion exchange mechanism responsible for C_P (see fig. 1).

The model is similar to that used by Manacher[16] and by Luyten, Rood and Tolhoek[17]. We take for the C's the following values: $C_V = 0.97 C_V^{(\beta)}$, $C_A = C_A^{(\beta)} = -1.25\, C_V^{(\beta)}$, $\mu C_M = 3.71\, C_V(\mu/m)$, $\mu C_P = 8\, C_A^{(\beta)}$, where $C_{V,A}^{(\beta)}$ are the constants of β-decay (in the usual notations $C_V^{(\beta)} = (1/\sqrt{2})G$), μ is the muon mass and m the proton mass. For the unknown parameter C_T we consider fours choices: $\mu C_T = 0$ (case I); $\mu C_T = 2(\mu/m)C_A$ (case II); $\mu C_T = 4(\mu/m)C_A$ (case III); $\mu C_T = C_A$ (case IV).

The values we find for the rate Λ of $\mu^- + p \rightarrow n + \nu$ and for the branching ratio R of the radiative mode $\mu^- + p \rightarrow n + \nu + \gamma$ relative to ordinary absorption on unpolarized proton, are reported in Table 1 (a_o is the mesic Bohr radius).

Table I

Predicted rate for ordinary absorption and branching ratio R for radiative absorption from unpolarized proton. The branching ratio $R(Ca^{40})$ for radiative absorption in Ca^{40} is reported in the last column. The pseudoscalar constant $\mu C_P(0)$ is put = $8C_A^{(\beta)}$, whereas C_T takes on different values in the four cases I - IV.

case	μC_T	$\Lambda_p / (\frac{\mu^2 C_V^{(\beta)}}{\pi^2 a_0^3})$	$R \times 10^4$	$R(Ca^{40}) \times 10^4$
I	0	4.80	4.50	2.12
II	$2(\mu/m)C_A$	4.62	4.96	2.40
III	$4(\mu/m)C_A$	4.56	5.44	2.65
IV	C_A	4.95	6.53	3.43

In fig. 2 we report the correctly normalized photon spectra $N(x)/\Lambda_p$ for the four cases I - IV ($x = k/\mu$, with k = photon momentum). One sees from Table I (and from fig. 2) that, whereas the ordinary absorption rate is rather insensitive to the tensor term C_T, the radiative absorption rate increases progressively with C_T (case IV gives R 45% larger than for case I).

3)- We shall now calculate the photon spectra and the branching ratio R for the practically interesting case of absorption from a nucleus. We first deal with ordinary absorption. The effective hamiltonian for ordinary μ^- absorption is easily derived from (1), neglecting terms depending on the nucleon velocity[18]. In the closure approximation, for nuclei with Z = A/2 and with complete proton and neutron shells, one finds for the total capture rate

$$\Lambda = \frac{Z^4}{\pi^2} \left(\frac{\mu}{137} \right)^3 \bar{\nu}^2 |\psi|^2 \, G^2 \left(1 - F(\bar{\nu})\right) \qquad (2)$$

where: $|\varphi|^2$ is the nuclear average of the square of the μ^- wave-function; $\bar{\nu}$ is the average neutrino energy. $F(\bar{\nu})$ is the ground state expectation value of $-(2/A)\ \sum_{ij} \tau_i^{(+)} \tau_j^{(-)} (\mathrm{sen}\,\bar{\nu} r_{ij})/(\bar{\nu} r_{ij})$ where i refers to the i-th nucleon, and r_{ij} is distance of the i-th from the j-th nucleon;

$$G^2 = G_v^2 + 3G_A^2 + G_P^2 - 2G_A G_P \qquad \text{with}$$

$$G_v = C_v\,(1 + \nu/2m) \tag{3}$$

$$G_A = C_A + C_T \frac{\nu^2}{2m} - C_v \frac{\nu}{2m}\,(1 + \mu_p - \mu_n) \tag{3'}$$

$$G_P = (\nu/2m)\left[-C_A + \mu C_P - C_v(1 + \mu_P - \mu_n) + \nu C_T\right] + \nu C_T \tag{3''}$$

In a Fermi model $F(\bar{\nu}) = 1 - \frac{3}{2}(\bar{\nu}/K_F) + \frac{1}{2}(\bar{\nu}/K_F)^3$ where K_F is the Fermi momentum. In the relevant region, $1-F \lesssim 0{,}22$[19], the Fermi model does not differ very much from more refined shell calculation[17]. On the other hand Λ is very sensitive to $\bar{\nu}$ (it roughly depends on $\bar{\nu}^3$). From the Telegdi value[19] $\bar{\nu}^2 G^2 = \pi \times 9{,}55 \times 10^{-30}\ \mathrm{sec}^{-1}\ \mathrm{cm}^3$ and with our choices of the C's we find approximately $\bar{\nu} \cong 0{,}80\ \mu = 84{,}5$ MeV[20]. The effective hamiltonian for radiative μ^- absorption can be derived from (1). We neglect the nucleon momenta and take the μ^- at rest. It is convenient to consider separately the emission of right - and left - circularly-polarized photons[21]. The polarization vectors are $\vec{\epsilon}_L = (1/2)^{1/2}\,(\vec{i} + i\vec{j})$ and $\vec{\epsilon}_R = \vec{\epsilon}_L^{\,*}$, with $\vec{i}$, $\vec{j}$, and $\vec{K}$ (photon momentum) forming a right handed frame. We write

$$H_{\mathrm{eff}}^{(L,R)} = \frac{1}{\sqrt{2}}\,(1 - \vec{\sigma}_e \cdot \vec{\nu}/\nu)\,\Omega_{L,R} \tag{4}$$

The expressions for Ω_L and Ω_R are given in the appendix. The photon spectrum is given by

$$N_{R,L}(K)dK = KdK \frac{2e^2}{(2\pi)^6}\left(\frac{Z\mu}{137}\right)^3 \sum_b |\varphi|^2 \int d\omega_\nu \, d\omega_K (\lambda - K)^2 \left| M_{R,L}\right|^2_{\nu = \lambda - K} \quad (5)$$

where $\lambda = \mu - \epsilon + E_a - E_b$ (ϵ = binding energy, E_a = ground state energy, E_b = energy of the excited state b), $|\varphi|^2$ is the average of the square of the μ^- wave function, $d\omega_\nu$ and $d\omega_K$ are the elements of solid angle around K and $\vec{\nu}$, and

$$\left|M_{R,L}\right|^2 := \frac{1}{4} \mathrm{Tr}(1 - \vec{\sigma}_\ell \cdot \vec{\nu}/\nu) / \langle b / \Omega_{R,L} e^{-i(\vec{\nu}+\vec{K})r_i} / a \rangle /^2 \quad (6)$$

with Tr acting on the lepton matrices. We consider nuclei with double closed shells and Z = A/2 and apply the closure approximation introducing an average $\langle\lambda\rangle = \mu - \epsilon + E_a - \langle E_b\rangle$. From $\nu + K = \langle\lambda\rangle$ one has $\langle\lambda\rangle$ = $= K_{max}$ and $\nu = K_{max} - K$. The total spectrum $N_R(K) + N_L(K)$ is proportional to

$$\sum_b \int d\omega_\nu \, d\omega_K \left[|M_R|^2 + |M_L|^2 \right]_{\nu = K_{max} - K} \quad (7)$$

For nuclei with double closed shells one can show that all nuclear matrix elements appearing in (7) can be reduced to the form $|\int 1|^2$ (see ref. 17). One finds that (7) can be written, after summing over b, as

$$(7) = 4\pi^2 \int_0^\pi \mathrm{sen}\theta d\theta \left\{ \alpha + \beta\cos\theta + \gamma\cos^2\theta + \delta\cos^3\theta \right\} \cdot$$
$$\cdot \langle a | \sum_{ij} \tau_i^{(+)} \tau_j^{(-)} e^{i(\vec{\nu}+\vec{K})\vec{r}_{ij}} | a \rangle_{\nu = K_{max} - K}$$

where $\cos\Theta = (\vec{\nu}\cdot\vec{K})/(\nu K)$, and α, β, γ and δ can be shown to be independent of θ to first order in μ/m. The ground state expectation value in the last equation can be expressed in terms of $F(|\vec{\nu}+\vec{K}|)$ with $\nu = K_{max} - K$, where F is the same as for ordinary capture (2). The nuclear structure effects in $N_{R,L}(x)/\Lambda$ are then all contained in the factor $[1 - F(\vec{\nu} + \vec{K})]/[1 - F(\vec{\nu})]$. In a Fermi model, considering that $\bar{\nu} \simeq K_{max} \simeq 85$ MeV and $K_F \simeq 250$ MeV, one can approximate such a factor with $(K_{max}/\bar{\nu})[(1-x)^2 + x^2 + 2x(1-x)\cos\theta]^{1/2}$ (where $x = K/K_{max}$). We expect that this result is not strongly dependent on the model. The photon spectra reported in Fig. 2 have been calculated for Ca^{40}, using $K_{max} = \bar{\nu}$, with the four choices I-IV for C_T. The ratio $R(Ca^{40})$ of radiative to ordinary absorption is given in Table I. Our result for $R(Ca^{40})$ in case I can be compared with the result by Rood and Tolhoek who find for slightly different parameters and a different model $R(Ca^{40}) = 2.29 \times 10^{-4}$. Considering the complexity of the calculation we find the agreement quite satisfactory. From the spectra of Fig. 3 and from table I we note again, as for the free proton case, the progressive increase of the radiative absorption with C_T. Also we see that, roughly, the ratio R on a nucleus is about one-half of the rate on unpolarized proton.

4) - We now consider the choices (i) $\mu C_P = 26 C_A$, $\mu C_T = 3.4\,(\mu/m) C_A$, and, (ii) $\mu C_P = 35\,C_A$, $\mu C_T = 3.4\,(\mu/m)\,C_A$, obtained by Huffaker and Greuling in their analysis of $B^{12} - N^{12}$ and μ-absorption[6]. (C_V, C_A and μC_M as before). The results are summarized in Table 2.

TABLE 2 - Predicted rate Λ for ordinary absorption and branching ratio R for radiative absorption from unpolarized proton. The branching ratio $R(Ca^{40})$ for radiative absorption in Ca^{40} is reported in the last column.

case	μC_P	μC_T	$\Lambda_p/(\frac{\mu^2 C_V^2 (\lambda)^2}{\pi^2 a_0^3})$	$R \times 10^4$	$R(Ca^{40}) \times 10^4$
(i)	$26 C_A$	$3.4(\mu/m)C_A$	5.14	12.8	6.65
(ii)	$35 C_A$	$3.4(\mu/m)C_A$	6.38	15.3	8.70

The photon spectra $N(X)/\Lambda$ for Ca^{40} for cases (i) and (ii) are reported in Fig. 4. The value $\overline{\nu} \simeq 0.80\ \mu$ can be shown to be a reasonable choice also for these cases; again we have assumed $K_{max} = \overline{\nu}$. The large radiative rate is essentially due to the large C_P. The maxima in the spectra are displaced to higher energy.

5) - In conclusion we would like to stress that the whole analysis inevitably depends on the approximate treatment of the nuclear dynamics, which makes it difficult to reach a definite conclusion. With such important provision, we can say that: (i) with the standard couplings (in particular $\mu C_P = 8C_A$) and an additional second class tensor coupling the experimental results seem to favor a value $C_T \gtrsim C_A$. Such an additional T coupling would be of help also in interpreting ordinary absorption in H_2. (ii) The sets of couplings(6) suggested from the B^{12} - N^{12} ft ratio together with μ^- capture in C^{12} apparently give a larger radiative rate than observed.

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REFERENCES. -

(1) - S. Weinberg, Phys. Rev. 112, 1375 (1958).

(2) - Y.K. Lee, L.W. Mo and C. Wu, Phys. Rev. Letters 10, 253 (1963).

(3) - M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964);
N. Cabibbo and R. Gatto, Nuovo Cimento 21, 872 (1962);
N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

(4) - T. R. Fisher, Phys. Rev. 130, 2388 (1963).

(5) - R. W. Peterson and N. W. Glass, Phys. Rev. 130, 292 (1963).

(6) - J. Huffaker and E. Greuling, Phys. Rev. 132, 738 (1963).

(7) - L. Wolfenstein, Nuovo Cimento 8, 882 (1958);
M. L. Goldberger and S. B. Treiman, Phys. Rev. 222, 354 (1958).

(8) - M. Morita and A. Fujii, Phys. Rev. 118, 606 (1960).

(9) - G. T. Reynolds, D. B. Scarl, R. A. Swanson, J. R. Waters and R. A. Zdanis, Phys. Rev. 129, 1790 (1963).

(10) - M. Conversi, R. Diebold and L. Di Lella, to be published.

(11) - A. Fujii, Nuovo Cimento 27, 1025 (1963);
E. Borchi and R. Gatto, to be published.

(12) - J. S. Bell and J. Lovseth, Nuovo Cimento 32, 433 (1964);
also J. Sens, Phys. Letters 9, 84 (1964).

(13) - W. Drechseler and B. Stech, to be published;
A. Fujii and Y. Yamaguchi, to be published in Progr. Theor. Phys.

(14) - See for instance: M. L. Younovich and V. S. Eveseev, E-1373, Dubna

(15) - R. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

(16) - G. K. Manacher, Carnegie Inst. of Techn. , Report NYO-9284 (1961).

(17) - J. R. Luyten, H. P. C. Rood and H. A. Tolhock, Nuclear Phys. 41, 236 (1963);
Our work is closely related to the work of these authors.

(18) - H. Primakoff, Revs. Mod. Phys. 31, 802 (1959);
see also reference (17).

(19) - V. L. Telegdi, Phys. Rev. Letters 8, 327 (1962).

(20) - From $1 - F(\bar{\nu}) = 0.22^{(19)}$ and with $K_F = 1.33 \times 10^{13}$ one finds directly $\bar{\nu} = 0.74\mu$, close to Primakoff[(18)] estimate $\bar{\nu} = 0.75\mu$.

(21) - R. E. Cutkosky, Phys. Rev. 107, 330 (1957);
Huang, Lee and Yang, Phys. Rev. 108, 1340 (1957).

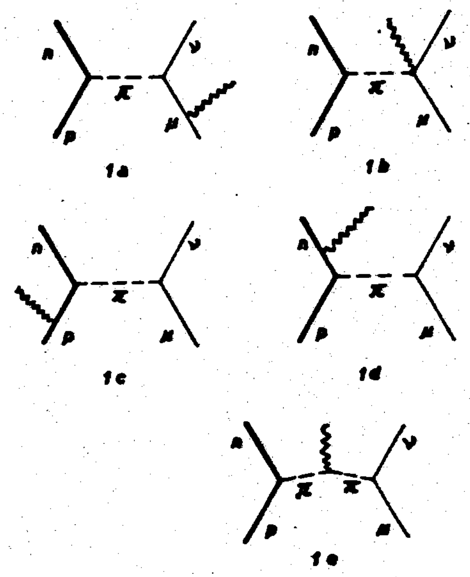

FIG. 1

Contributions to $\mu^- + p \rightarrow n + \nu + \gamma$ proportional to the pseudoscalar coupling C_P.

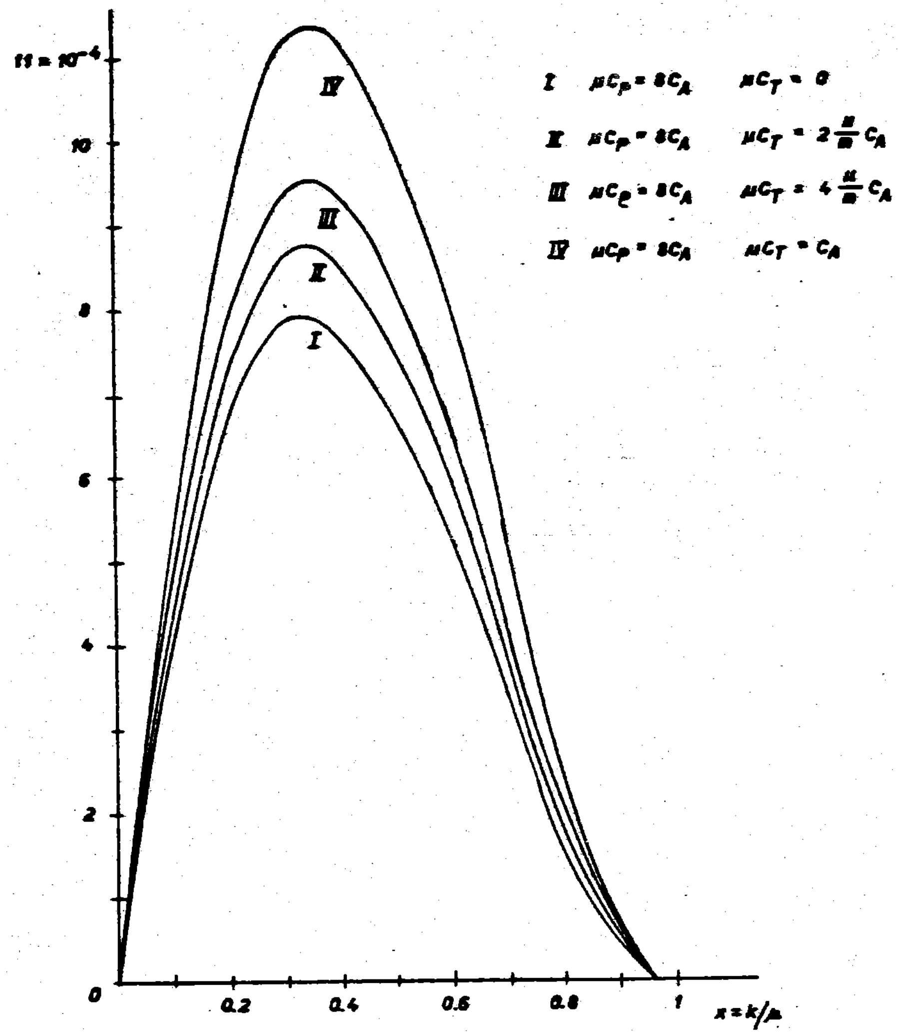

FIG. 2

Photon spectra $N(x)/\Lambda_p$ for radiative muon absorption by an unpolarized proton for cases I - IV.

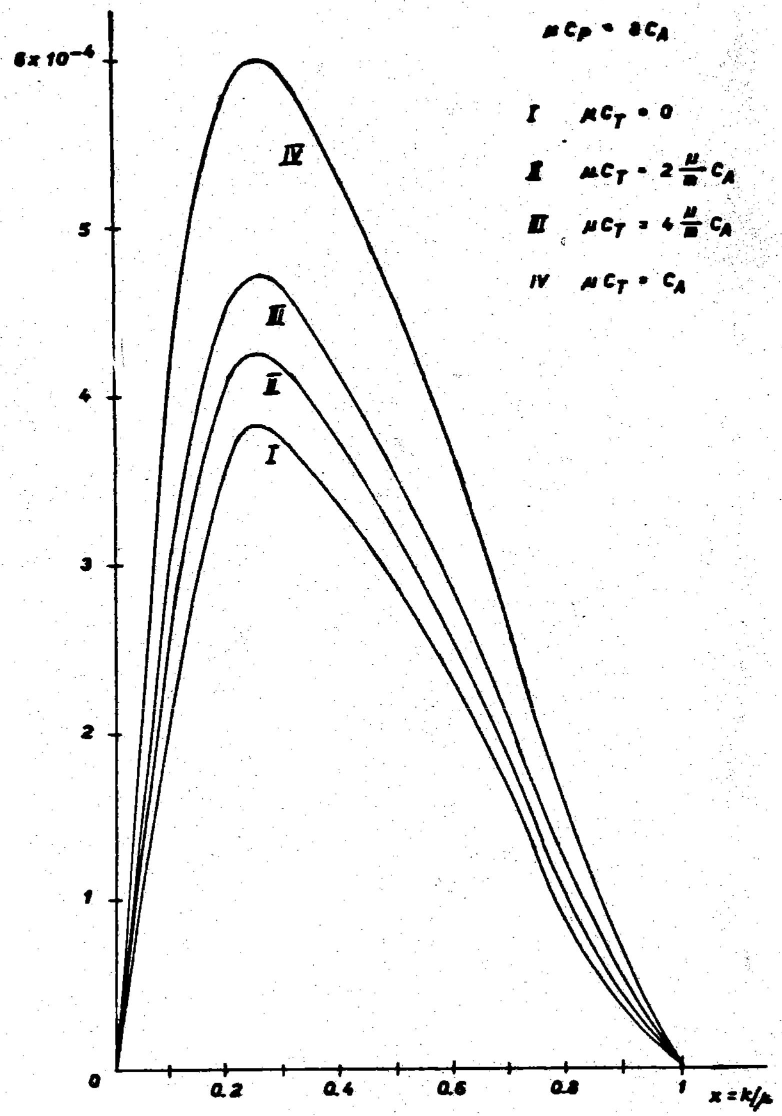

FIG. 3

Photon spectra $N(x)/\Lambda$ for radiative muon absorption by Ca^{40} for cases I - IV.

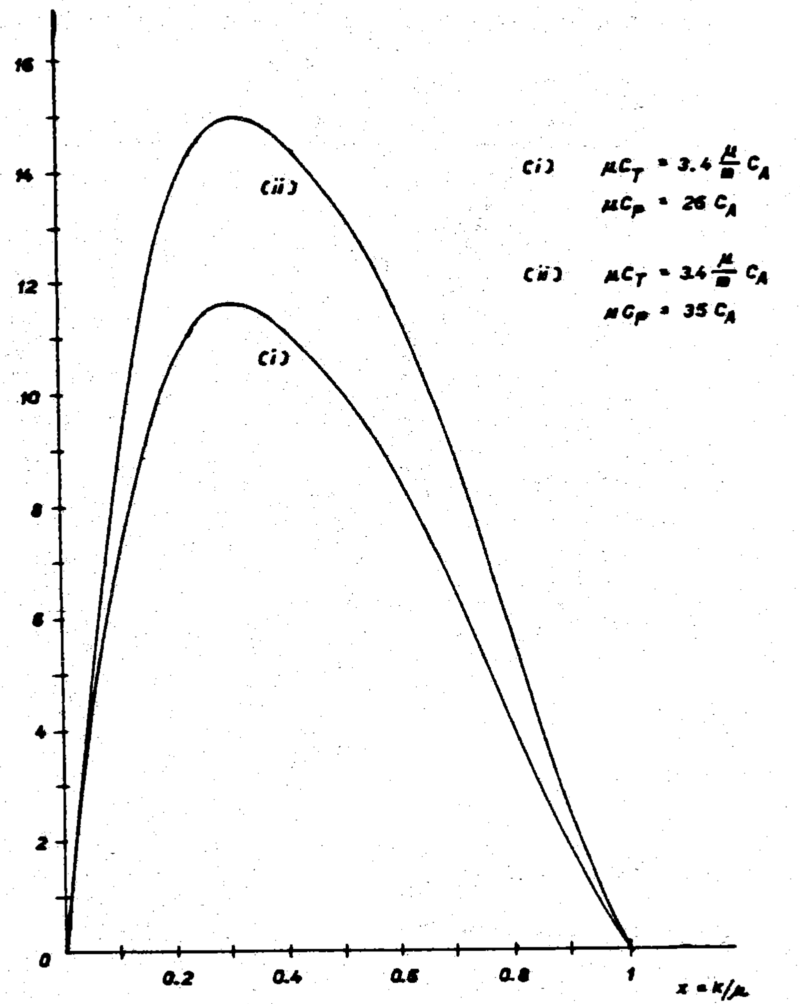

FIG. 4

Photon spectra $N(x)/\Lambda$ for radiative muon absorption by Ca^{40} for case (i) and (ii).

APPENDI X -

The effective hamiltonian for μ^- radiative absorption by a nucleus is given by Eq. 4 with:

$$\Omega_L = \frac{1}{2m}\sum_i \tau_i^{(-)}\Big\{-\Big[2mC_T + C_V + C_A + (\mu_p-\mu_n)C_V + \nu C_T + \mu C_P^{(N)} +$$

$$+ \frac{K}{2m}\mu C_P^{(N)}(\mu_p-\mu_n+1)\Big](\vec{\sigma}_i\cdot\vec{\epsilon}) - (\mu_p-\mu_n)C_T(\vec{\nu}\cdot\vec{\epsilon}) +$$

$$+ \frac{2\mu C_P^{(N)}}{(\vec{\nu}+\vec{k})^2+\mu^2}(\epsilon\cdot\vec{\nu})(\vec{\sigma}_i\cdot[\vec{k}+\vec{\nu}]) - \frac{\mu C_P^{(N)}}{2m}(\mu_p-\mu_n+1)(\vec{\sigma}_i\cdot\vec{\nu})(\vec{\sigma}_i\cdot\vec{\epsilon}) +$$

$$+ (\vec{\sigma}^L\cdot\vec{\epsilon})\Big[(C_P^{(L)}-C_T)(\vec{\nu}\cdot\vec{\sigma}_i) + (C_P^{(L)} + \frac{2m}{K}C_M)(\vec{\sigma}_i\cdot\vec{K}) +$$

$$+ (\mu_p-\mu_n)(C_A + kC_T) - kC_T\Big] + (C_T - \frac{2m}{k}C_M)(\vec{\sigma}^L\cdot\vec{k})(\vec{\sigma}_i\cdot\vec{\epsilon}) +$$

$$- i(1+\mu_p-\mu_n)C_M(\vec{\sigma}^L\cdot\vec{\nu}\wedge\vec{\epsilon}) + (\vec{\sigma}^L\cdot\vec{\sigma}_i)\Big[kC_T(\vec{\sigma}_i\cdot\vec{\epsilon}) -$$

$$- C_M(\vec{\nu}\cdot\vec{\epsilon})\Big]\Big\}\,\delta(\vec{r}-\vec{r}_i)$$

$$\Omega_R = \frac{1}{2m}\sum_i \tau_i^{(-)}\Big\{\Big[-2mC_T + (\mu_p-\mu_n)C_V - \mu C_P^{(N)} + C_V - C_A + \frac{\nu-K}{\mu}(2mC_M + C_V) -$$

$$- \nu C_T - \frac{(\vec{\nu}+\vec{k})^2}{\mu}C_T + \frac{k\nu}{\mu}C_T - \frac{k\mu C_P^{(N)}}{2m}(\mu_p-\mu_n+1) - \frac{2m}{\mu}C_A\Big](\vec{\sigma}_i\cdot\vec{\epsilon}) +$$

$$+\Big[\frac{2\nu C_T}{\mu} - (\mu_p-\mu_n)C_M\Big](\vec{\nu}\cdot\vec{\epsilon}) + \Big[\frac{C_V}{\mu} + \frac{\mu}{2m}C_P^{(N)}(\mu_p-\mu_n+1) + \frac{2m}{\mu}C_M -$$

$$- \frac{\nu C_T}{\mu}\Big](\vec{\sigma}_i\cdot\vec{\nu})(\vec{\sigma}_i\cdot\vec{\epsilon}) + \Big(\frac{C_T}{\mu} + \frac{2\mu C_P^{(N)}}{(\vec{\nu}+\vec{k})^2+\mu^2}\,(\vec{\nu}\cdot\vec{\epsilon})(\vec{\sigma}_i\cdot[\vec{k}+\vec{\nu}]\Big) +$$

$$+ (\vec{\sigma}^L\cdot\vec{\epsilon})\Big[-\frac{2m}{\mu}(C_V + C_A) - kC_T(\mu_p-\mu_n-1) + (\mu_p-\mu_n)C_A - \frac{kC_V}{\mu} +$$

$$+ 2mC_M + \Big(\frac{C_A}{\mu} - \frac{2m}{\mu}C_T\Big)(\vec{\sigma}_i[\vec{\nu}+\vec{k}] + (\vec{\sigma}^L\cdot k)\Big[\Big(-\frac{2m}{\mu}C_M - \frac{C_V}{\mu} +$$

$$+ C_T - \frac{\kappa C_T}{\mu}\Big](\vec{\sigma}_i \cdot \vec{\epsilon}) + \frac{C_T}{\mu}(\vec{\sigma}_i \cdot \vec{\nu})(\vec{\sigma}_i \cdot \vec{\epsilon})\Big] + i\left(\frac{C_V}{\mu} - C_T\right)(\vec{\sigma}^L \cdot \vec{\nu} \wedge \vec{\epsilon}) +$$

$$+ (\vec{\sigma}^L \cdot \sigma_i)\Big[\left(\frac{2m}{\mu} C_M + \frac{C_V}{\mu} - C_T\right)(\vec{\nu} \cdot \vec{\epsilon}) + \Big(-2m C_M - C_V + C_A +$$

$$+ \frac{2m}{\mu} C_A - (\kappa + \nu) C_T\Big)(\vec{\sigma}_i \cdot \vec{\epsilon}) + \frac{C_T}{\mu}(\vec{\sigma}_i \cdot \vec{\nu})(\vec{\nu} \cdot \vec{\epsilon}) +$$

$$+ \frac{C_T}{\mu}(\vec{\sigma}_i \cdot \vec{k})(\vec{\nu} \cdot \vec{\epsilon})\Big]\Big\}\, \delta(\vec{r} - \vec{r}_i)$$

where

$$C_P^{(N)} = C_P \frac{\mu^2 + m_\pi^2}{\vec{\nu}^2 - \vec{k}^2 + m_\pi^2} \qquad \text{and} \qquad C_P^{(L)} = C_P \frac{\mu^2 + m_\pi^2}{(\vec{\nu} + \vec{k})^2 + m_\pi^2}$$